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## REGULAR HADAMARD MATRIX OF ORDER 196 AND SIMILAR MATRICES

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**Purpose:** This note discusses two level quasi-orthogonal matrices which were first highlighted by J. J. Sylvester; Hadamard matrices, symmetric conference matrices, and weighing matrices are the best known of these matrices with entries from the unit disk. The goal of this note is to develop a theory of such matrices based on preliminary research results. **Methods:** Our new regular Hadamard matrix constructed for order 196, suggests a source of ideas to construct regular Hadamard matrices of orders  $n = 1 + p \times q = 1 + p \times (1 + 2m)$ , where  $p, q$  are twin odd integer ( $q - p = 2$ );  $m = (q - 1)/2$ , prime, order of inner blocks. **Results:** We present a new method aimed to give regular Hadamard matrix of order 196 and similar matrices. Such kinds of regular Hadamard matrix of order 36 were done by Jennifer Seberry (1969), that inspired to find matrices of orders  $4k^2$ ,  $k$  integer, 36, 100, 196, ..., 1444 and many others. We apply this result to the family of regular matrices obtaining a new infinite family of Cretan matrices with orders  $4t + 1$ ,  $t$  an integer, 37, 101, 197, ..., 1445, etc. **Practical relevance:** Web addresses are given for other illustrations and other matrices with similar properties. Algorithms to construct regular matrices have been implemented in developing software of the research program-complex.

**Keywords** – Quasi-Orthogonal Matrices, Hadamard Matrices, Regular Hadamard Matrices, Cretan Matrices, Legendre Symbols.

**AMS Subject Classification:** 05B20; 20B20.

We present a new method aimed to give regular Hadamard matrices, that can be used to construct Cretan matrices [1, 2] with orders  $4t + 1$ ,  $t$  is an integer. Similar kinds of regular Hadamard matrix of order 36 were done by Jennifer Seberry (1969) [3] that inspired to find matrices of orders  $4k^2$ ,  $k$  integer, 36, 100, 196, and many others. The conditions for the existence request SBIBD is given in [4]. We observe an example of regular Hadamard matrix, order 196.

Let order of regular Hadamard matrix is  $n = 1 + p \times q$ ,  $p$  is prime (or prime power),  $q$  is prime or not prime. Thus we take composition  $n = 1 + p \times q = 1 + p \times (1 + 2m)$ , where  $p, q$  are twin odd integer ( $q - p = 2$ );  $m = (q - 1)/2$  is prime and it is order of blocks of the following two-border structure

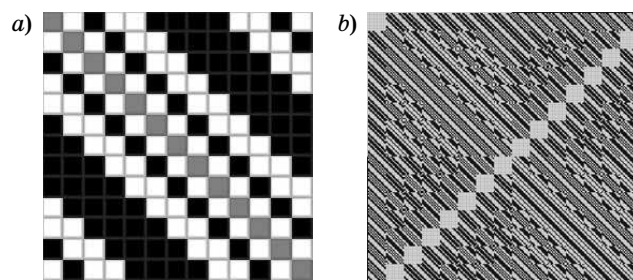
$$H = \begin{pmatrix} 1 & e^T & e^T & \dots & e^T & -e^T & \dots & -e^T \\ e & J & B & \dots & B & A & \dots & A \\ e & B^T & & & & & & \\ \vdots & \vdots & & C_{11} & & & & C_{12} \\ e & B^T & & & & & & \\ -e & A^T & & & & & & \\ \vdots & \vdots & & C_{21} & & & & C_{22} \\ -e & A^T & & & & & & \end{pmatrix},$$

here  $A, B$  of size  $p$ ,  $J, e$  are the same size matrix and vector of all 1s respectively; matrix border

has  $m$  blocks  $B, \dots, B$  and  $m$  blocks  $A, \dots, A$ ; and  $C_{11}, C_{12}, C_{21}, C_{22}$  are  $m \times m$  matrices of blocks of a core consisted  $J, A, B$ ; "T" — sign of transposition.

For order  $n = 196 = 1 + 13 \times 15 = 1 + 13 \times (1 + 2 \times 7)$  we have  $p = 13, q = 15$  is not prime,  $m = 7$  is prime. Let be  $C_{11} = \text{circ}(A, -B, -B, A, -B, A, A)$  be a circulant matrix of order  $m = 7$  of Legendre symbols where "1" (and "0"), "-1" changed to  $A, -B$  respectively; a complementary matrix is  $C_{22} = \text{circ}(-B, A, A, -B, A, -B, -B)$ ;  $C_{21} = C_{12}^T$ , and  $C_{12} = \text{circback}(-A, -A, B, -A, B, B, J)$  is the back-circulant matrix of Legendre symbols, taken in reversed order, where "0" is changed by  $J$ .

Then if  $A = I - Q, B = -I - Q; Q$  is a circulant matrix of order  $p = 13$  of Legendre symbols (Figure, a), Hadamard matrix of order 196 has sums of all columns and rows equal to 14 (i.e. it is regular



■ Circulant matrix  $Q$  of Legendre symbols (a) and regular Hadamard matrix of order 196 (b)

matrix) (Figure, *b*). In such a way, we can get set of regular Hadamard matrices orders  $36 = 1 + 5 \times 7 = 1 + 5 \times (1 + 2 \times 3)$  and it can be constructed with one border and one circulant core due 7 is prime,  $100 = 1 + 9 \times 11 = 1 + 9 \times (1 + 2 \times 5)$  — this matrix has a special cell-structure due order of  $\mathbf{Q}$  is  $9 = 3 \times 3$ ,  $196 = 1 + 13 \times 15 = 1 + 13 \times (1 + 2 \times 7)$  is given matrix, ...,  $1444 = 1 + 37 \times 39 = 1 + 37 \times (1 + 2 \times 19)$  it is used as a test with positive result,

and many others with the same form described above. We apply this result to the family of regular matrices obtaining a new infinite family of *Cretan matrices* of *Fermat-type* [1, 2] with orders  $4t + 1$ ,  $t$  an integer, 37, 101, 197, ..., 1445, etc. which will be studied in later papers. We acknowledge the use of the <http://www.mathscinet.ru> and <http://www.wolframalpha.com> sites for the number and symbol calculations in this paper.

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