

## A NEW CONSTRUCTION FOR HADAMARD MATRICES<sup>1</sup>

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An *Hadamard matrix*  $H$  is a square matrix of *ones* and *minus ones* whose row (and hence column) vectors are orthogonal. The order  $n$  of an Hadamard matrix is necessarily 1, 2 or  $4t$  with  $t=1, 2, 3, \dots$ . It has been conjectured that this condition ( $n=1, 2$  or  $4t$ ) also insures the existence of an Hadamard matrix. Constructions have been given for particular values of  $n$  and even for various infinite classes of values. While other constructions exist, those given by [1]–[7] exhaust the previously known values of  $n$ . This paper gives a new construction which yields, among others, the previously unknown value  $n=156$ , leaving only two undecided values of  $n=4t \leq 200$  (these are 116 and 188).

An Hadamard matrix is said to be of the *Williamson type* if it has the structure imposed by Williamson [6], that is

$$H = \begin{vmatrix} A & B & C & D \\ -B & A & -D & C \\ -C & D & A & -B \\ -D & -C & B & A \end{vmatrix},$$

where each of  $A, B, C, D$  is a symmetric circulant  $t \times t$  matrix. Notice that if a Williamson type matrix exists for  $n=4t$ , then an Hadamard matrix (not obviously Williamson) of order  $m=12t$  would exist provided one could find a  $12 \times 12$  matrix with the following properties. Each row and column must contain precisely three  $\pm A$ 's, three  $\pm B$ 's, three  $\pm C$ 's, three  $\pm D$ 's and the rows must be formally orthogonal (i.e.,  $A, B, C, D$  are to be considered as independent quantities). We have discovered such a matrix and display it as Figure 1.

Among the *known* orders of Williamson type matrices [1], [6], only 52 yields a new value of  $n$  by this construction. This gives an Hadamard matrix of order 156. For definiteness, the first rows of  $A, B, C, D$  for one of the Williamson type Hadamard matrices of order 52 are given (here  $+$  means  $+1$  and  $-$  stands for  $-1$ ).

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		1	2	3	4	5	6	7	8	9	10	11	12	13
	A	+	+	-	-	+	-	+	+	-	+	-	-	+
	B	+	-	-	-	+	+	+	+	+	+	-	-	-
	C	+	+	+	-	+	+	-	-	+	+	-	+	+
	D	+	+	-	+	-	+	+	+	+	-	+	-	+
H =	A	A	A	A	B	-B	C	-C	-D	B	C	-D	-D	
	A	-A	B	-A	-B	-D	D	-C	-B	-D	-C	-C		
	A	-B	-A	A	-D	D	-B	B	-C	-D	C	-C		
	B	A	-A	-A	D	D	D	C	C	-B	-B	-C		
	B	-D	D	D	A	A	A	C	-C	B	-C	B		
	B	C	-D	D	A	-A	C	-A	-D	C	B	-B		
	D	-C	B	-B	A	-C	-A	A	B	C	D	-D		
	-C	-D	-C	-D	C	A	-A	-A	-D	B	-B	-B		
	D	-C	-B	-B	-B	C	C	-D	A	A	A	D		
	-D	-B	C	C	C	B	B	-D	A	-A	D	-A		
	C	-B	-C	C	D	-B	-D	-B	A	-D	-A	A		
	-C	-D	-D	C	-C	-B	B	B	D	A	-A	-A		

FIGURE 1

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